

# Penn Cambria Curriculum

<b>Course Name</b>	<b>Calculus</b>
<b>Length of Course</b>	<i>1 credit (1 period per day for one semester)- Weighted at 1.10 for GPA calculations</i>
<b>Grade Level</b>	12 ** Dual Enrollment Course
<b>Prerequisites</b>	85% or higher in Trig/Pre-Calculus
<b>Course Description</b>	<p>This is a rigorous and advanced course in the basic calculus sequence intended for students with mathematics, engineering, science, business, or statistics career goals. This course is designed to provide the opportunity for students, who so elect, to extend their mathematical knowledge beyond the current high school academic curriculum. This course gives students the opportunities to systematically and informally investigate the central ideas of calculus that contribute to a deepening of their understanding of function and its utility in representing and answering questions about real-world phenomena.</p> <p><i>It may be taken as a dual enrollment course through Penn Highlands Community College for college credit.</i></p>
<b>Units of Study</b>	Preparation for Calculus Limits and Their Properties Differentiation Applications of Differentiation Integration Logarithmic, Exponential, and other Transcendental Functions
<b>Materials</b>	<p><b>Text:</b> <u>Calculus of a Single Variable</u> – Larson/Hostetler/Edwards – Houghton Mifflin c2002</p> <p><b>Supplemental Materials:</b> Graphing Calculator</p>

\*\* Standards Alignment is based on PDE's revised draft standards dated January 2010 provided by the Standards Aligned System ([www.pdesas.org](http://www.pdesas.org)). Keystone Assessment Anchor alignment is included at the end of the curriculum document.

## Unit 1: Preparation for Calculus

**Estimated Time:** 7 Days

### Standard Alignment:

- 2.1.11. B – Use factoring to create equivalent forms of polynomials.
- 2.1.11. F – Understand the concepts of exponential and logarithmic forms and use the inverse relationships between exponential and logarithmic expression to determine unknown quantities in equations.
- 2.3.11. E – Describe how a change in the value of one variable in a formula affects the value of the measurement.
- 2.5.11. A – Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.
- 2.5.11. B – Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas, and results.
- 2.6.11. E – Make predictions based on lines of best fit or draw conclusions on the value of a variable in a population based on the results of a sample.
- 2.8.11. B – Evaluate and simplify algebraic expressions and solve and graph linear, quadratic, exponential, and logarithmic equations and inequalities, and solve and graph systems of equations and inequalities.
- 2.8.11. D – Demonstrate an understanding and apply properties of functions (domain, range, inverses) and characteristics of families of functions (linear, polynomial, rational, trigonometric, exponential, logarithmic).

### Curricular Objectives:

#### A. Graphs and Models

- a. Sketch the graph of an equation by point plotting
- b. Plot the x and y intercept of a graph
- c. Test a graph for symmetry in the x-axis, y-axis, and the origin
- d. Solve systems of equations algebraically, graphically, and by using technology
- e. Use technology to graph equations
- f. Interpret mathematical models for real-life data

#### B. Linear Models and Rates of Change

- a. Find the slope of a line given two points, a linear equation, or a graph
- b. Write a linear equation given a point and slope
- c. Interpret slope as a rate of change in a real-life application
- d. Sketch the graph of a linear equation in slope-intercept form
- e. Write equations of lines that are parallel or perpendicular to a given line

#### C. Functions and their Graphs

- a. Use function notation to represent and evaluate a function
- b. Find the domain and range of a function
- c. Sketch the graph of a function
- d. Identify different types of transformations and reflections of functions
- e. Classify functions and recognize combinations of functions

#### D. Fitting Models to Data

- a. Fit a linear model to a real-life data set
- b. Fit a quadratic model to a real-life data set

### Assessments/ Measurement of Objectives:

- Objective quizzes/tests
- Graded homework/classroom exercises

### **Suggested Methods of Instruction / Learning Activities:**

- Lecture
- Computer-aided instruction
- Guided practice
- Drill and practice
- Peer-to-peer instruction/tutoring/discussion
- Class Discussion

<h2><b>Unit 2: Limits and Their Properties</b></h2>
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**Estimated Time:** 13 Days

### **Standard Alignment:**

- 2.1.11. A – Model and compare values of irrational and complex numbers.
- 2.1.11. B – Use factoring to create equivalent forms of polynomials.
- 2.1.11. D – Use exponential, scientific, and calculator notation to represent any rational number.
- 2.2.11. C – Evaluate numerical expressions that include the four basic operations and operations of powers and roots, reciprocals, opposites, and absolute values.
- 2.3.11. E – Describe how a change in the value of one variable in a formula affects the value of the measurement.
- 2.5.11. A – Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.
- 2.5.11. B – Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
- 2.8.11. B - Evaluate and simplify algebraic expressions and solve and graph linear, quadratic, exponential, and logarithmic equations and inequalities, and solve and graph systems of equations and inequalities.
- 2.8.11. D – Demonstrate an understanding and apply properties of functions (domain, range, inverses) and characteristics of families of functions (linear, polynomial, rational, trigonometric, exponential, logarithmic).
- 2.8.11.E – Use combinations of symbols and numbers to create expressions, equalities, and inequalities in two or more variables, systems of equations and inequalities, and functional relationships that model problem situations.
- 2.8.11. F – Interpret the results of solving equations, inequalities, systems of equations, and inequalities in the context of the situation that motivated the model.

### **Curricular Objectives:**

#### **A. A Preview of Calculus**

- a. Explain what calculus is and how it compares to pre-calculus
- b. Explain that the tangent line problem is basic to calculus
- c. Explain that the area problem is basic to calculus

#### **B. Finding Limits Graphically and Numerically**

- a. Estimate a limit using a graphical or numerical approach
- b. Describe different ways a limit fails to exist

#### **C. Evaluating Limits Analytically**

- a. Evaluate a limit using the properties of limits
- b. Develop and use a strategy for finding limits
- c. Evaluate a limit using dividing out and rationalizing techniques
- d. Evaluate a limit using the squeeze theorem

**D. Continuity and One-Sided Limits**

- a. Determine the continuity at a point and continuity on an open interval
- b. Determine one-sided limits and continuity on a closed interval
- c. Use properties of continuity
- d. State and apply the Intermediate Value Theorem

**E. Infinite Limits**

- a. Determine infinite limits from the left and from the right
- b. Find and sketch the vertical asymptotes of a function

**Assessments/ Measurement of Objectives:**

- Objective quizzes/tests
- Graded homework/classroom exercises

**Suggested Methods of Instruction / Learning Activities:**

- Lecture
- Computer-aided instruction
- Guided practice
- Drill and practice
- Peer-to-peer instruction/tutoring/discussion
- Computer simulations
- Class Discussion

**Unit 3: Differentiation**

**Estimated Time:** 17 Days

**Standard Alignment:**

- 2.1.11. A – Model and compare values of irrational and complex numbers.
- 2.1.11. B - Use factoring to create equivalent forms of polynomials.
- 2.1.11. D – Use exponential, scientific, and calculator notation to represent any rational number.
- 2.2.11. C – Evaluate numerical expressions that include the four basic operations and operations of powers and roots, reciprocals, opposites, and absolute values.
- 2.3.11. E – Describe how a change in the value of one variable in a formula affects the value of the measurement.
- 2.5.11. A – Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.
- 2.5.11. B – Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
- 2.8.11. B - Evaluate and simplify algebraic expressions and solve and graph linear, quadratic, exponential, and logarithmic equations and inequalities, and solve and graph systems of equations and inequalities.
- 2.8.11. D – Demonstrate an understanding and apply properties of functions (domain, range, inverses) and characteristics of families of functions (linear, polynomial, rational, trigonometric, exponential, logarithmic).
- 2.8.11.E – Use combinations of symbols and numbers to create expressions, equalities, and inequalities in two or more variables, systems of equations and inequalities, and functional relationships that model problem situations.
- 2.8.11. F – Interpret the results of solving equations, inequalities, systems of equations, and inequalities in the context of the situation that motivated the model.

**Curricular Objectives:****A. The Derivative and the Tangent Line Problem**

- a. Find the slope of the tangent line to a curve at a point
- b. Use the limit definition to find the derivative of a function
- c. Explain the relationship between differentiability and continuity

**B. Differentiation Rules and Rates of Change**

- a. Find the derivative of a function using the Constant Rule
- b. Find the derivative of a function using the Power Rule
- c. Find the derivative of a function using the Constant Multiple Rule
- d. Find the derivative of a function using the Sum and Difference Rules
- e. Find the derivative of the sine function and of the cosine function
- f. Use derivatives to solve problems involving rates of change

**C. The Product and Quotient Rules and Higher-Order Derivatives**

- a. Find the derivative of a function using the Product Rule
- b. Find the derivative of a function using the Quotient Rule
- c. Find the derivative of a trigonometric function
- d. Find a higher-order derivative
- e. Use derivatives to solve problems dealing with velocity and acceleration

**D. The Chain Rule**

- a. Find the derivative of a composite function using the Chain Rule
- b. Find the derivative of a function using the General Power Rule
- c. Simplify the derivative of a function using algebra
- d. Find the derivative of a trig function using the Chain Rule

**E. Implicit Differentiation**

- a. Distinguish between functions written in implicit form and explicit form
- b. Use implicit differentiation to find the derivative of a function

**F. Related Rates**

- a. Find a related rate
- b. Use related rates to solve real-life problems

**Assessments/ Measurement of Objectives:**

- Objective quizzes/tests
- Graded homework/classroom exercises

**Suggested Methods of Instruction / Learning Activities:**

- Lecture
- Computer-aided instruction
- Guided practice
- Drill and practice
- Computer simulations
- Peer-to-peer instruction/tutoring/discussion
- Class Discussion

## Unit 4: Applications of Differentiation

**Estimated Time:** 18 Days

### Standard Alignment:

- 2.1.11. A – Model and compare values of irrational and complex numbers.
- 2.1.11. B - Use factoring to create equivalent forms of polynomials.
- 2.1.11. D – Use exponential, scientific, and calculator notation to represent any rational number.
- 2.2.11. C – Evaluate numerical expressions that include the four basic operations and operations of powers and roots, reciprocals, opposites, and absolute values.
- 2.3.11. E – Describe how a change in the value of one variable in a formula affects the value of the measurement.
- 2.5.11. A – Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.
- 2.5.11. B – Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
- 2.8.11. B - Evaluate and simplify algebraic expressions and solve and graph linear, quadratic, exponential, and logarithmic equations and inequalities, and solve and graph systems of equations and inequalities.
- 2.8.11. D – Demonstrate an understanding and apply properties of functions (domain, range, inverses) and characteristics of families of functions (linear, polynomial, rational, trigonometric, exponential, logarithmic).
- 2.8.11.E – Use combinations of symbols and numbers to create expressions, equalities, and inequalities in two or more variables, systems of equations and inequalities, and functional relationships that model problem situations.
- 2.8.11. F – Interpret the results of solving equations, inequalities, systems of equations, and inequalities in the context of the situation that motivated the model.
- 2.11.11.A –Determine and interpret maximum and minimum values of a function over a specified interval.
- 2.11.11.B –Analyze and interpret rates of growth/decay.

### Curricular Objectives:

#### A. Applications of Differentiation

- a. Understand the definition of extrema on an interval
- b. Understand the definition of relative extrema of a function on an open interval
- c. Find extrema on a closed interval

#### B. Rolle's Theorem and the Mean Value Theorem

- a. Explain and use Rolle's Theorem
- b. Explain and use the Mean Value Theorem

#### C. Increasing and Decreasing Functions and the First Derivative Test

- a. Determine the intervals on which a function is increasing or decreasing
- b. Apply the First Derivative Test to find relative extrema of a function

#### D. Concavity and the Second Derivative Test

- a. Determine intervals on which a function is concave upward or concave downward
- b. Find any points of inflection of the graph of a function
- c. Apply the Second Derivative Test to find relative extrema of a function

#### E. Limits at Infinity

- a. Determine finite limits at infinity
- b. Determine the horizontal asymptotes, if any, of the graph of a function
- c. Determine infinite limits at infinity

#### F. A Summary of Curve Sketching

- a. Analyze and sketch the graph of a function

#### G. Optimization

- a. Solve applies minimum and maximum problems

#### H. Newton's Method

- a. Approximate the zero of a function using Newton's Method

## **I. Differentials**

- a. Demonstrate the concept of a tangent line approximation
- b. Compare the value of the differential,  $dy$ , with the actual change in  $y$ ,  $\Delta y$
- c. Estimate a propagated error using a differential
- d. Find the differential of a function using differentiation formulas

### **Assessments/ Measurement of Objectives:**

- Objective quizzes/tests
- Graded homework/classroom exercises

### **Suggested Methods of Instruction / Learning Activities:**

- Lecture
- Computer-aided instruction
- Guided practice
- Drill and practice
- Computer simulations
- Peer-to-peer instruction/tutoring/discussion
- Class Discussion

<h2><b>Unit 5: Integration</b></h2>
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**Estimated Time:** 18 Days

### **Standard Alignment:**

- 2.1.11. A – Model and compare values of irrational and complex numbers.
- 2.1.11. B - Use factoring to create equivalent forms of polynomials.
- 2.1.11. D – Use exponential, scientific, and calculator notation to represent any rational number.
- 2.2.11. C – Evaluate numerical expressions that include the four basic operations and operations of powers and roots, reciprocals, opposites, and absolute values.
- 2.3.11. E – Describe how a change in the value of one variable in a formula affects the value of the measurement.
- 2.5.11. A – Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.
- 2.5.11. B – Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
- 2.8.11. B - Evaluate and simplify algebraic expressions and solve and graph linear, quadratic, exponential, and logarithmic equations and inequalities, and solve and graph systems of equations and inequalities.
- 2.8.11. C – Recognize, describe and generalize patterns, using sequences and series to predict long-term outcomes.
- 2.8.11. D – Demonstrate an understanding and apply properties of functions (domain, range, inverses) and characteristics of families of functions (linear, polynomial, rational, trigonometric, exponential, logarithmic).
- 2.8.11.E – Use combinations of symbols and numbers to create expressions, equalities, and inequalities in two or more variables, systems of equations and inequalities, and functional relationships that model problem situations.
- 2.8.11. F – Interpret the results of solving equations, inequalities, systems of equations, and inequalities in the context of the situation that motivated the model.
- 2.11.11.C- Estimate areas under curves using sums of areas.

### **Curricular Objectives:**

#### **A. Antiderivatives and Indefinite Integration**

- a. Write the general solution of a differential equation

- b. Use indefinite integral notation for antiderivatives
- c. Use basic integration rules to find antiderivatives
- d. Find a particular solution of a differential equation

**B. Area**

- a. Use sigma notation to write and evaluate a sum
- b. Explain the concept of area
- c. Approximate the area of a plane region
- d. Find the area of a plane region using limits

**C. Riemann Sums and the Definite Integral**

- a. Describe the definition of a Riemann sum
- b. Evaluate a definite integral using limits
- c. Evaluate a definite integral using properties of definite integrals

**D. The Fundamental Theorem of Calculus**

- a. Evaluate a definite integral using the Fundamental Theorem of Calculus
- b. Explain and use the Mean Value Theorem for integrals
- c. Find the average value of a function over a closed interval
- d. Explain and use the Second Fundamental Theorem of Calculus

**E. Integration by Substitution**

- a. Use pattern recognition to evaluate an indefinite integral
- b. Use a change of variables to evaluate an indefinite integral
- c. Use the General Power Rule for integration to evaluate an indefinite integral
- d. Use a change of variables to evaluate a definite integral

**F. Numeric Integration**

- a. Approximate a definite integral using the Trapezoidal Rule
- b. Approximate a definite integral using Simpson's Rule

**Assessments/ Measurement of Objectives:**

- Objective quizzes/tests
- Graded homework/classroom exercises

**Suggested Methods of Instruction / Learning Activities:**

- Lecture
- Computer-aided instruction
- Guided practice
- Drill and practice
- Computer simulations
- Peer-to-peer instruction/tutoring/discussion
- Class Discussion



## Unit 6: Logarithmic, Exponential, and Other Transcendental Functions

**Estimated Time:** 17 Days

### Standard Alignment:

- 2.1.11. A – Model and compare values of irrational and complex numbers.
- 2.1.11. D – Use exponential, scientific, and calculator notation to represent any rational number.
- 2.1.11. F – Understand the concepts of exponential and logarithmic forms and use the inverse relationships between exponential and logarithmic expression to determine unknown quantities in equations.
- 2.2.11. C – Evaluate numerical expressions that include the four basic operations and operations of powers and roots, reciprocals, opposites, and absolute values.
- 2.3.11. E – Describe how a change in the value of one variable in a formula affects the value of the measurement.
- 2.5.11. A – Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.
- 2.5.11. B – Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
- 2.8.11. B - Evaluate and simplify algebraic expressions and solve and graph linear, quadratic, exponential, and logarithmic equations and inequalities, and solve and graph systems of equations and inequalities.
- 2.8.11. D – Demonstrate an understanding and apply properties of functions (domain, range, inverses) and characteristics of families of functions (linear, polynomial, rational, trigonometric, exponential, logarithmic).
- 2.8.11.E – Use combinations of symbols and numbers to create expressions, equalities, and inequalities in two or more variables, systems of equations and inequalities, and functional relationships that model problem situations.
- 2.10.11.A –Identify, create, and solve practical problems involving right triangles using the trigonometric functions and the Pythagorean Theorem.
- 2.11.11.A –Determine and interpret maximum and minimum values of a function over a specified interval.
- 2.11.11.B –Analyze and interpret rates of growth/decay.
- 2.11.11.C- Estimate areas under curves using sums of areas.

### Curricular Objectives:

#### A. The Natural Logarithmic Function: Differentiation

- a. Develop and use properties of the natural logarithmic function
- b. Explain the definition of the number  $e$
- c. Find derivatives of functions involving the natural logarithmic function

#### B. The Natural Logarithmic Function: Integration

- a. Use the Log Rule for integration to integrate a natural function
- b. Integrate trigonometric functions

#### C. Inverse Functions

- a. Verify that one function is the inverse of another function
- b. Determine if a function has an inverse function
- c. Find the derivative of an inverse function

#### D. Exponential Functions: Differentiation and Integration

- a. Develop properties of the natural exponential function
- b. Differentiate natural exponential functions
- c. Integrate natural exponential functions

#### E. Bases Other than $e$ and Applications

- a. Define exponential functions that have bases other than  $e$
- b. Differentiate and integrate functions that have bases other than  $e$
- c. Use exponential functions to model compound interest and exponential growth

#### F. Differential Equations: Growth and Decay

- a. Use separation of variables to solve a simple differential equation
- b. Use exponential functions to model growth and decay

**G. Differential Equations: Separation of Variables**

- a. Use initial conditions to find particular solutions to differential equations
- b. Recognize and solve differential equations that can be solved by separation of variables
- c. Recognize and solve homogeneous differential equations

**H. Inverse Trigonometric Functions: Differentiation and Integration**

- a. Develop the properties of the six inverse trigonometric functions
- b. Differentiate an inverse trigonometric function
- c. Integrate an inverse trigonometric function

**Assessments/ Measurement of Objectives:**

- Objective quizzes/tests
- Graded homework/classroom exercises

**Suggested Methods of Instruction / Learning Activities:**

- Lecture
- Computer-aided instruction
- Guided practice
- Drill and practice
- Computer simulations
- Peer-to-peer instruction/tutoring/discussion
- Class Discussion